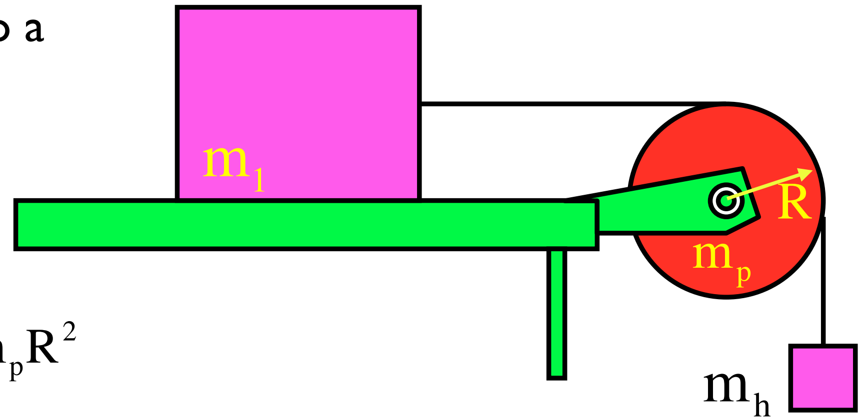
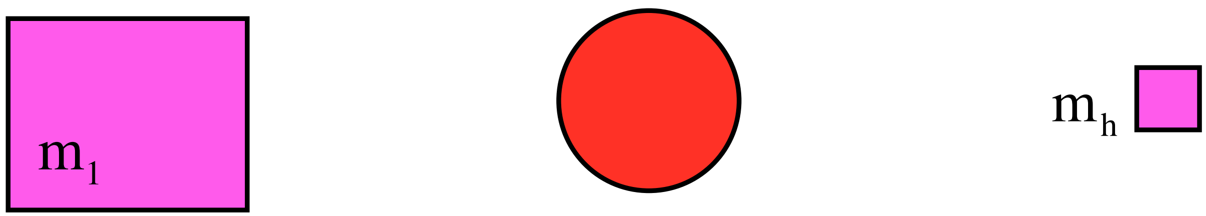
5.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on an frictional tabletop. Your finger perpendicular to the radius vector and a distance R/3 from the axis of rotation maintains motionlessness. Known is:



$$m_1, m_h, m_p, R, g, \mu_k, \text{ and } I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$$

a.) Draw a f.b.d. identifying all the forces acting on both masses and the pulley.



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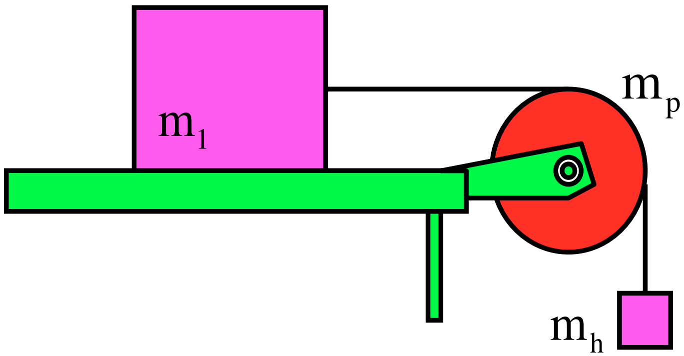
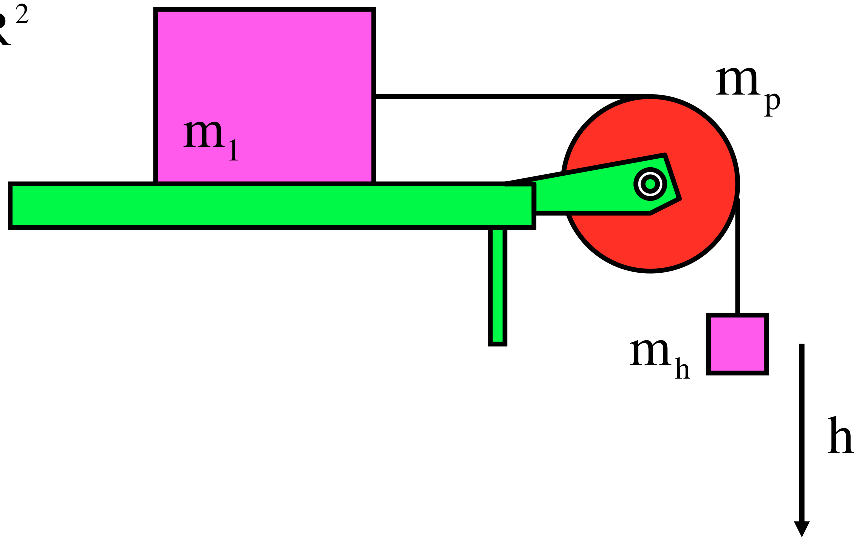


b.) Derive an expression for the system's accelerate.

c.) What is the pulley's *angular acceleration*?

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \mu_k,\ \text{and}\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$

d.) The hanging mass drops a distance "h." What is its *velocity* magnitude at that point?

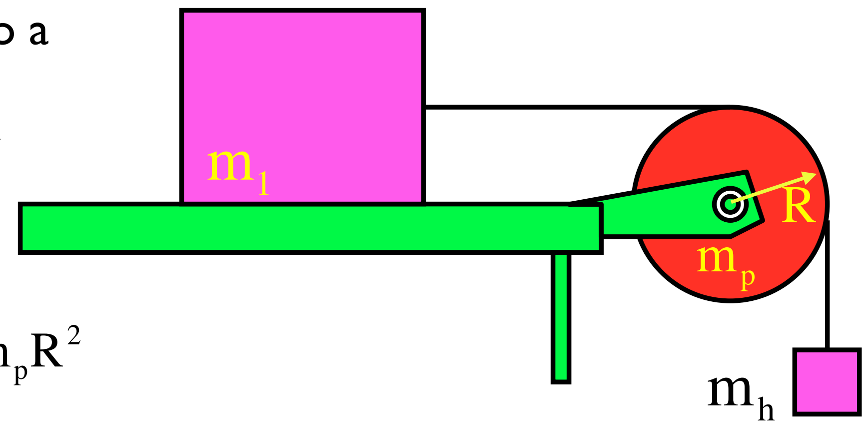


e.) The hanging mass drops a distance "h." What is the pulley's *angular velocity* at that point?

f.) The hanging mass drops a distance "h." What is the pulley's *angular momentum* at that point?

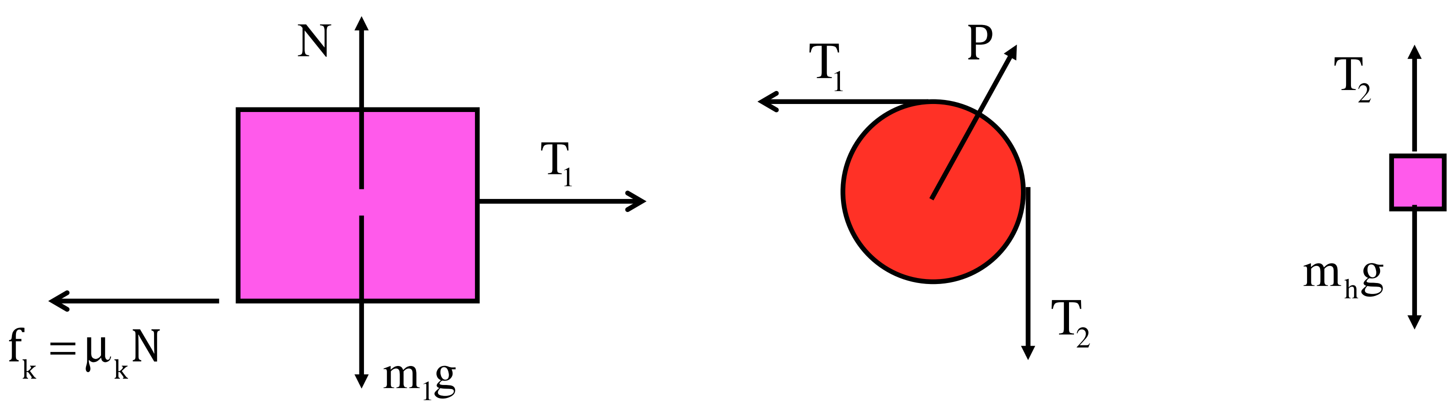
g.) For amusement, determine the *moment of inertia* about an axis R/3 units from the pulley's axis of rotation.

5.) A hanging mass is attached to a string which is threaded over a massive pulley and attached to a second mass sitting on an frictional tabletop. Your finger perpendicular to the radius vector and a distance R/3 from the axis of rotation maintains motionlessness. Known is:



$$m_1,\ m_h,\ m_p,\ R,\ g,\ \mu_k,\ \text{and}\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$

a.) Draw a f.b.d. identifying all the forces acting on both masses and the pulley.



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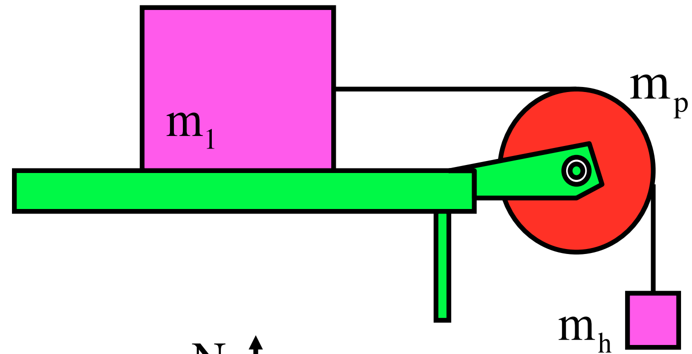
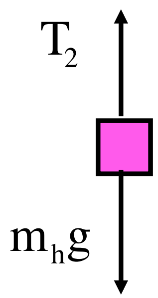


f.b.d. on hanging mass

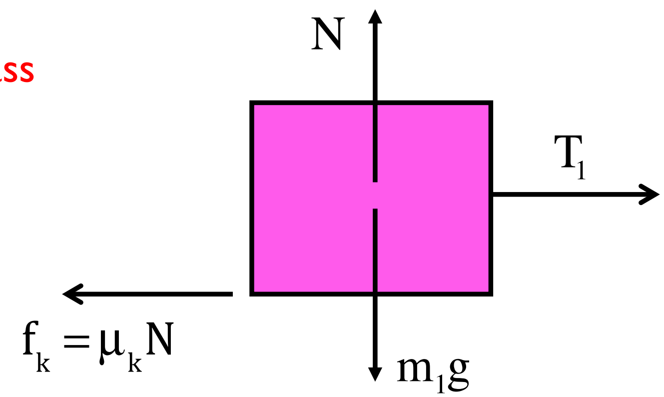


$$\sum F_{m_h,y}$$

$$T_2 - m_h g = -m_h a$$

$$\Rightarrow \quad T_2 = m_h g - m_h a$$

f.b.d. on tabletop mass



$$\sum F_{m_1,y}$$

$$N - m_1 g = m_1 a_y \quad (a_y \to 0)$$

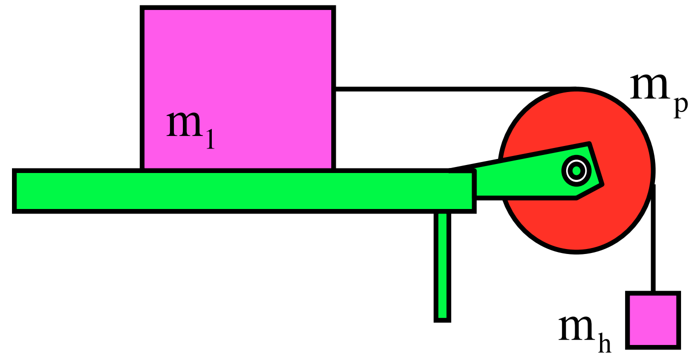
$$\Rightarrow \quad N = m_1 g$$

$$\sum F_{m_1,x}$$

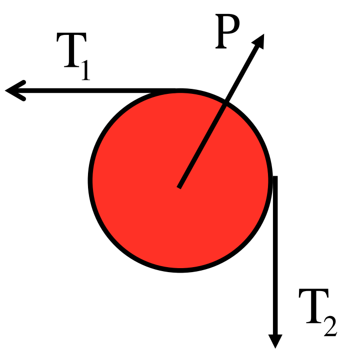
$$T_1 - \mu_k N = m_1 a$$

$$\Rightarrow \quad T_1 = \mu_k (m_1 g) + m_1 a$$

$$m_1,\ m_h,\ m_p,\ R,\ g,\ \mu_k,\ \text{and}\ I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$



f.b.d. on pulley



$$\sum \Gamma_{\text{pulley}}$$

$$-T_2R + T_1R = -I_{pin}\alpha$$

$$\Rightarrow\ -T_2\not{R} + T_1\not{R} = -\left(\frac{1}{2}m_p\not{R}^2\right)\left(\frac{a}{\not{R}}\right)$$

$$\Rightarrow\ -T_2 + T_1 = -\frac{1}{2}m_pa$$

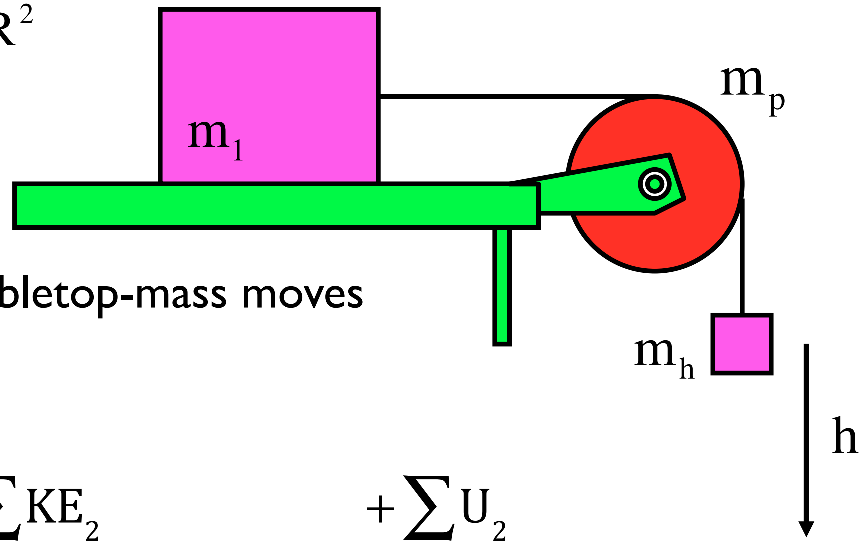
$$\Rightarrow\ -\left(m_hg - m_ha\right) + \left(\mu_k m_1 g + m_1 a\right) = -\frac{1}{2}m_pa$$

$$\Rightarrow\ a = \frac{-m_hg + \mu_k m_1 g}{-\left(\frac{1}{2}m_p + m_1 + m_h\right)}$$

c.) What is the pulley's *angular acceleration?* $\alpha = \left(\frac{a}{R}\right)$ (good enough!)

$m_1,\ m_h,\ m_p,\ R,\ g,\ \mu_k$, and $I_{\text{cm of pulley}} = \frac{1}{2} m_p R^2$

d.) The hanging mass drops a distance "h."
What is its *velocity* magnitude at that point?



Note that as the hanging mass drops, the tabletop-mass moves to the right the same distance.

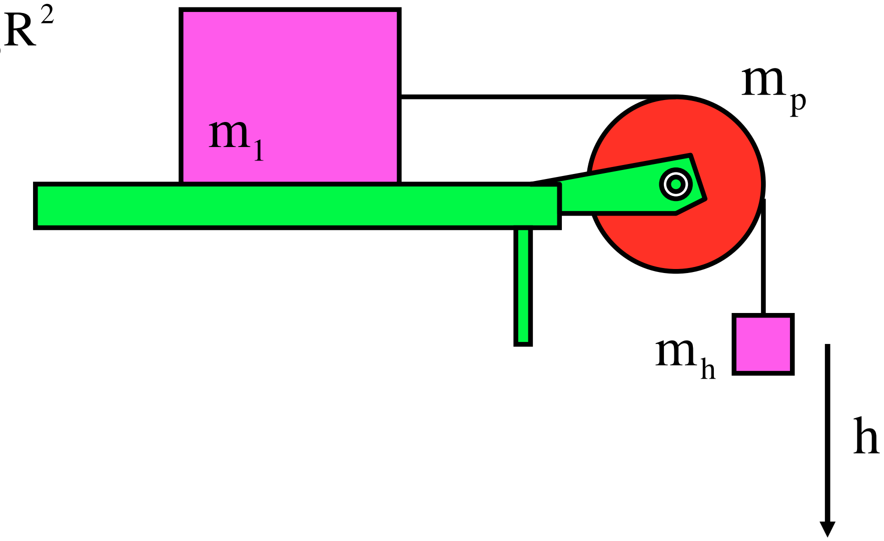
$$\sum KE_1 \quad \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 \quad + \left[m_h g h\right] + \left(-fh\right) = \left[\frac{1}{2}m_1 v^2 + \frac{1}{2}m_h v^2 + \frac{1}{2}I_{pulley}\omega^2\right] + \quad 0$$

$$\Rightarrow \quad \left[m_h g h\right] - \left[\left(\mu_k m_1 g\right)\left(h\right)\right] = \left[\frac{1}{2}m_1 v^2 + \frac{1}{2}m_h v^2 + \frac{1}{2}\left(\frac{1}{2}m_p R^2\right)\left(\frac{v}{R}\right)^2\right]$$

$$\Rightarrow \quad v = \sqrt{\frac{2\left(m_h g h - \left(\mu_k m_1 g\right)\left(h\right)\right)}{m_1 + m_h + \frac{m_p}{2}}}$$

$$m_1, m_h, m_p, R, g, \mu_k, \text{ and } I_{\text{cm of pulley}} = \frac{1}{2}m_pR^2$$



e.) The hanging mass drops a distance "h." What is the pulley's *angular velocity* at that point?

$$\omega = \frac{v}{R}$$

f.) The hanging mass drops a distance "h." What is the pulley's *angular momentum* at that point?

$$L = I_{pin}\omega$$

g.) For amusement, determine the *moment of inertia* about an axis R/3 units from the pulley's axis of rotation.

Using the parallel axis theorem, we get:

$$\begin{aligned} I_p &= I_{cm} + md^2 \\ &= \frac{1}{2}m_pR^2 + m_p\left(\frac{R}{3}\right)^2 \\ &= \frac{11}{18}m_pR^2 \end{aligned}$$